Decrypting the (slow) nonlinearity in ENSO observations: potential for skillful predictions

Shivsai Ajit Dixit

Indian Institute of Tropical Meteorology (IITM), Pune

AND

Prof. B. N. Goswami

Cotton University, Guwahati

IV International Conference on El Nino Southern Oscillation: ENSO in a warmer Climate, Guayaquil, Ecuador.

Date: 16th October 2018

ENSO is a nonlinear phenomenon

- Nonlinearity of the ocean, atmosphere and the coupled system
- Multi-scale character – spatial and temporal
- “Fast” and “Slow” nonlinearities
- “Fast” = climate noise etc. which can be modeled as stochastic processes
- “Slow” = essentially having the timescale comparable to or larger than the dominant ENSO periodicity and cannot therefore be modeled as a stochastic process

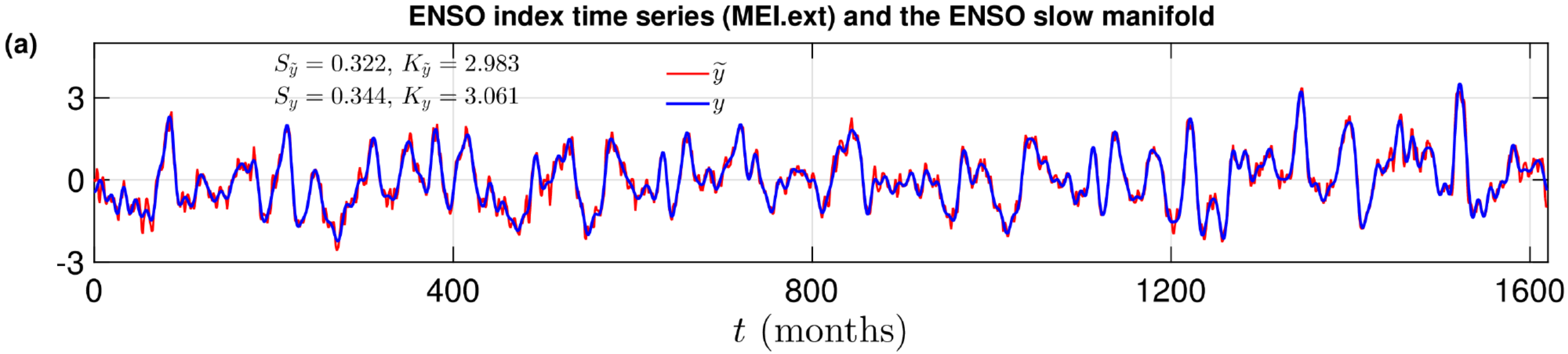
(for example, see Penland and Magorian 1993; Penland 2007; McPhaden 2015; Levine and McPhaden 2016)

ENSO is a nonlinear phenomenon … contd.

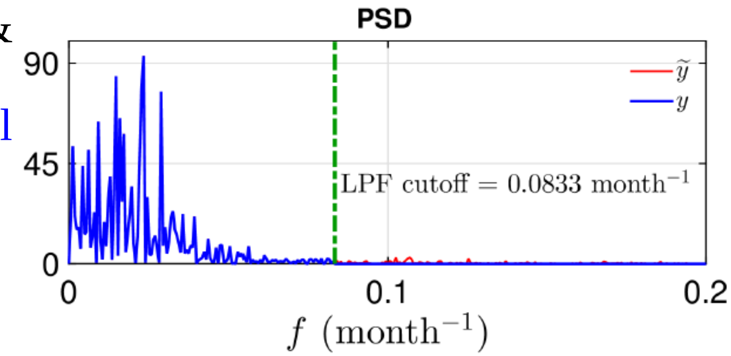
- We present our analysis “ENSO Observations” to uncover the “Slow” nonlinear map hidden in the observations
- Our approach is based on ideas of nonlinear dynamics
- State-space embedding of a monthly ENSO monitoring index to reconstruct ENSO dynamics in higher-dimensional state space
- We have used time delay embedding in a 5-dimensional space

(see also Elsner and Tsonis 1992; Bauer and Brown 1992; Elsner and Tsonis 1993; Tsonis et al. 1993)

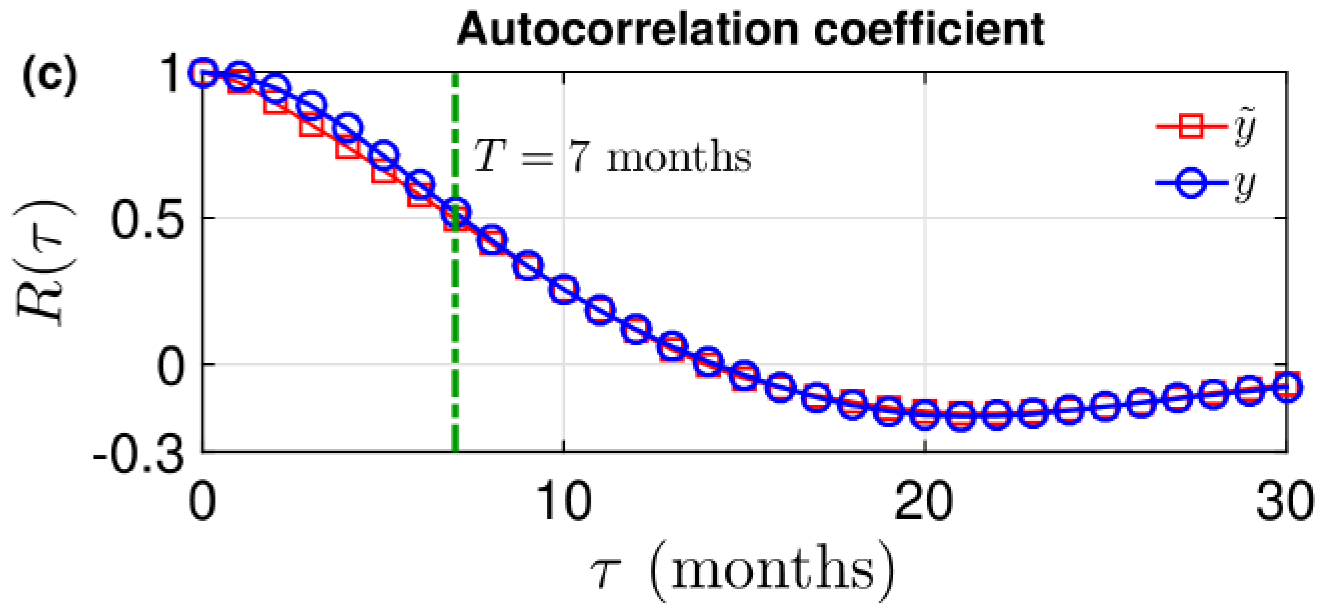
ENSO index timeseries captures temporal nonlinearity



- MEI.ext – Coupled ENSO index, first PC of SST and SLP fields from COADS, monthly index values from 18 (Wolter & Timlin 2011),
- Captures essential features of ENSO phenomenon such as El Nino - La Nina asymmetry (An and Jin 2004)
- Climate noise – high frequency components varying faster than 1 year!
- MEI.ext – Climate noise = Slow manifold *y* of ENSO
- Note that the Slow manifold retains > 95% of signal energy and also all essential features of the ENSO phenomenon



Integral timescale and dimensionless time



$$T = \int_{\tau=0}^{\tau=14} R(\tau)\mathrm{d}\tau = 7 \text{ months}$$

$$\hat{t} = t/T,\, \hat{\tau} = \tau/T$$

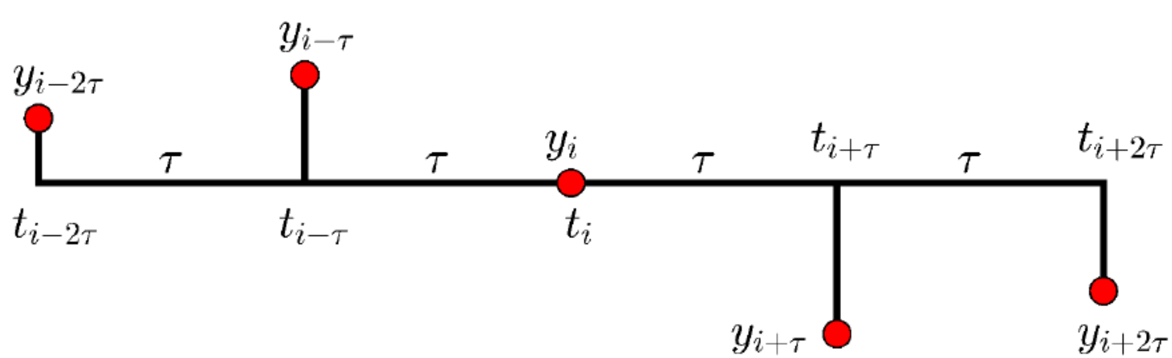
State-space embedding of the "Slow Manifold" signal *y*

Coordinates of the state space for sample delay of 2 months

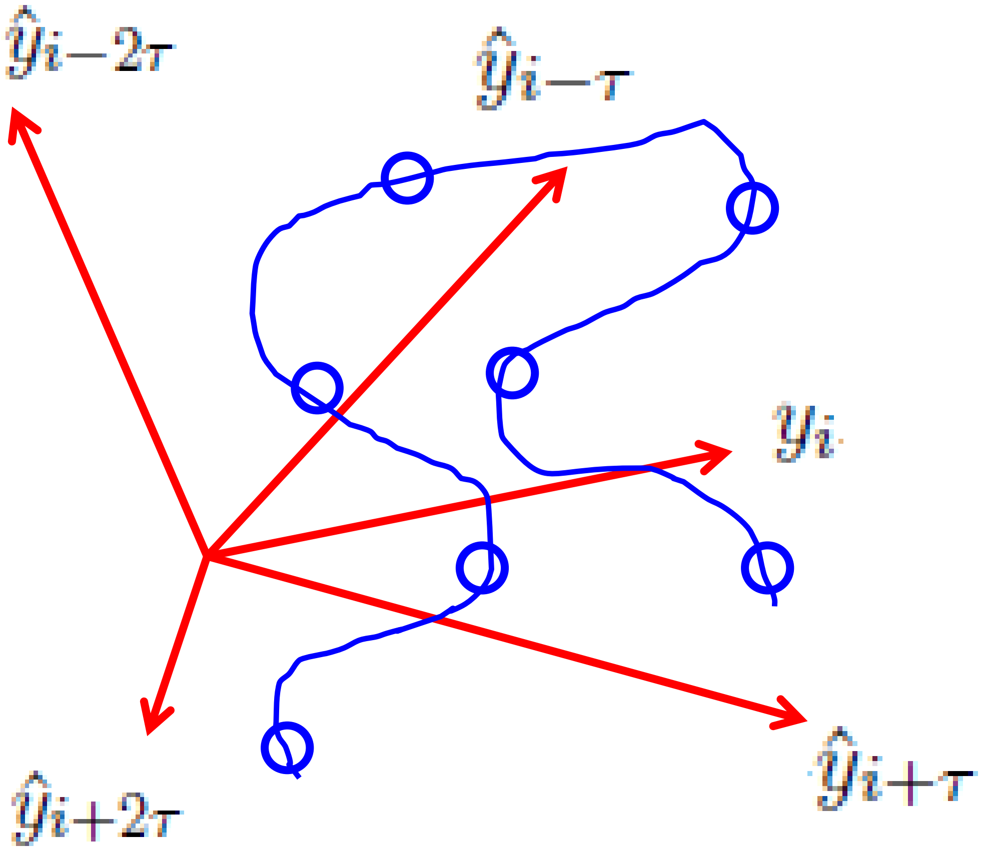
Each row represents a state point in the state space and all rows represent the trajectory

y1	y3	y5	y7	y9
y2	y4	y6	y8	y10
..	..	..	..	..
yn-8	yn-6	yn-4	yn-2	yn

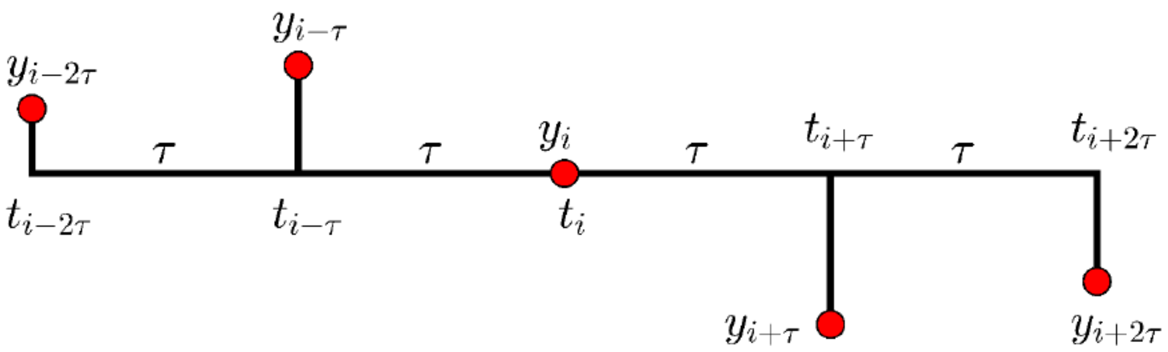
Five-point stencil



State-space trajectory in 5 dimensions



Time invariant map



A relationship that holds between the coordinates for ALL times

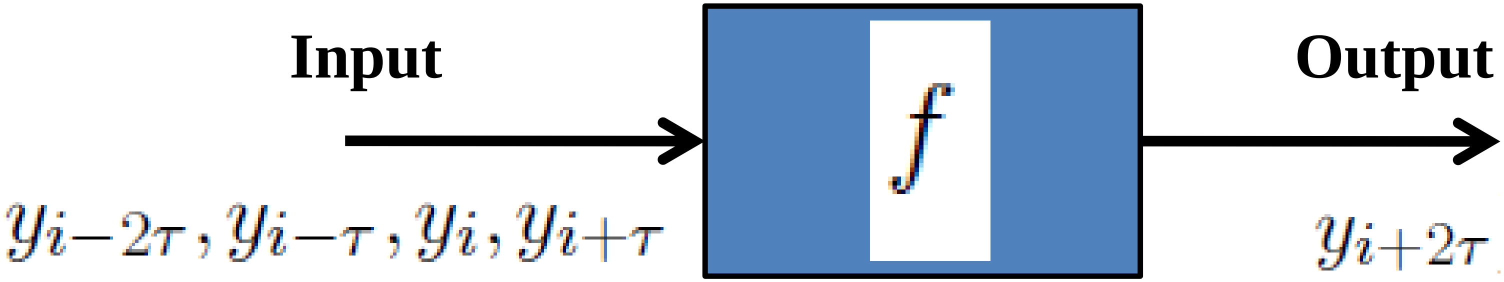
$$f\left(y_{i-2\tau}, y_{i-\tau}, y_i, y_{i+\tau}, y_{i+2\tau}\right) = 0.$$

f **could be a deterministic and nonlinear function**

Generalized Volterra Series

$$f\left(y_{i-2\tau}, y_{i-\tau}, y_i, y_{i+\tau}, y_{i+2\tau}\right) = 0.$$

Input-output relationship generalized power series (Flake 1963)



$$a_0 + \sum_{m=i-2\tau}^{m=i+2\tau} a_m y_m + \sum_{m=i-2\tau}^{m=i+2\tau} \sum_{n=i-2\tau}^{n=i+2\tau} b_{mn} y_m y_n + \cdots = 0$$

Which terms to retain? Some Guidelines

$$a_0 + \sum_{m=i-2\tau}^{m=i+2\tau} a_m y_m + \sum_{m=i-2\tau}^{m=i+2\tau} \sum_{n=i-2\tau}^{n=i+2\tau} b_{mn} y_m y_n + \cdots = 0$$

1. The fundamental structure of dominant nonlinearity in most geophysical systems is *quadratic* in nature arising from advective processes. Therefore quadratic terms in Eq. A2 may be expected to model dominant *slow* nonlinearity of the ENSO.

2. If one wishes to use Eq. A2 for prediction of $y_{i+2\tau}$, then the ~~terms nonlinear in $y_{i+2\tau}$~~ need to be left out for unambiguous predictions.

Guidelines … Contd.

3. If one rearranges Eq. A2, for prediction purposes, such that $y_{i+2\tau}$ is on the left side and everything else is moved to the right side, the denominator on the right side should not tend to or become zero. **Add a DC shift to *y*** $\hat{y} = y + S$

4. The total number of terms (excluding a_0) in Eq. A2 may be restricted to three, if possible, to facilitate "visualization" (by three-dimensional plotting) of the extent to which Eq. A2 is getting satisfied.

$$F\left(\hat{y}_{i-2\tau}, \hat{y}_{i-\tau}, \hat{y}_i, \hat{y}_{i+\tau}, \hat{y}_{i+2\tau}\right) = 0,$$

$$A_0 + \sum_{m=i-2\tau}^{m=i+2\tau} A_m \hat{y}_m + \sum_{m=i-2\tau}^{m=i+2\tau} \sum_{n=i-2\tau}^{n=i+2\tau} B_{mn} \hat{y}_m \hat{y}_n + \cdots = 0$$

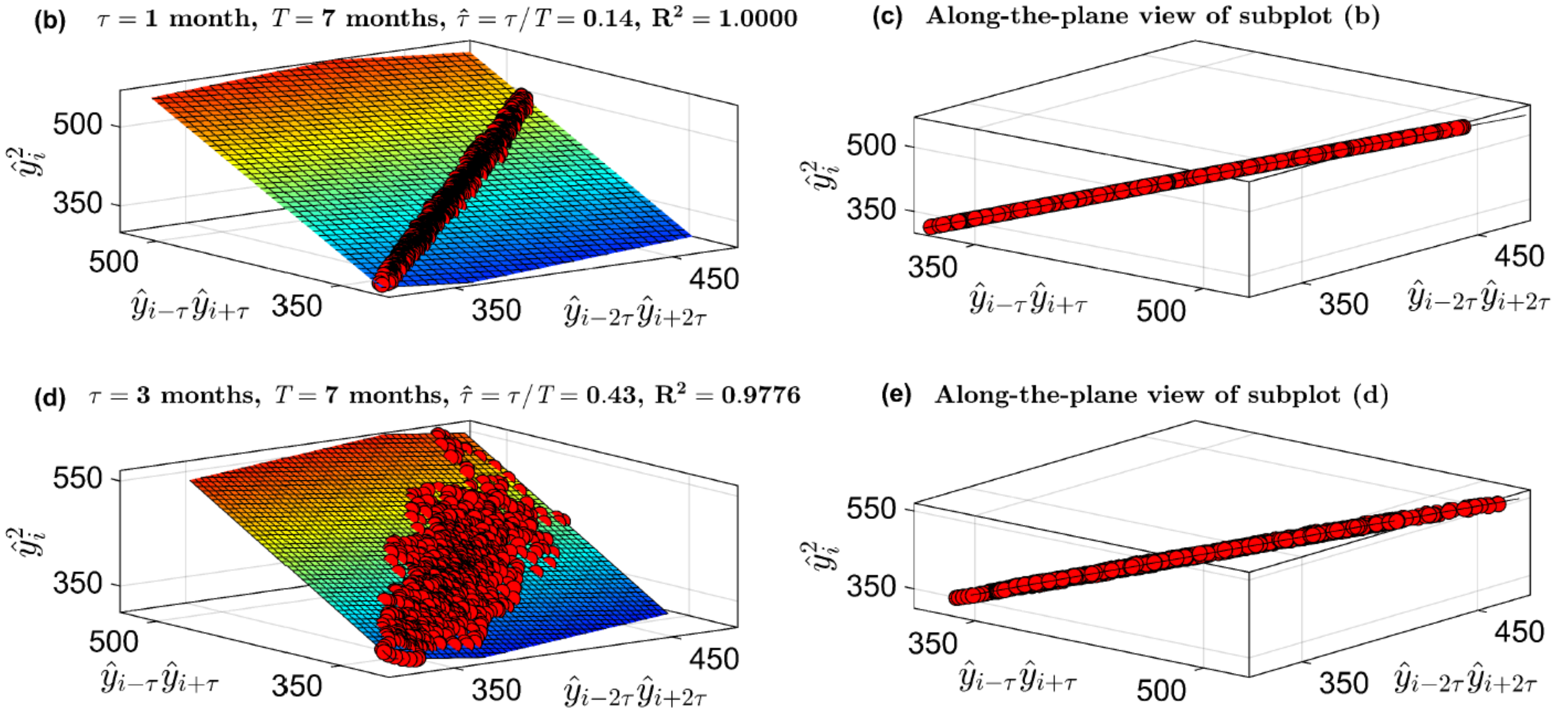
The map after several trials – Serendipity!!

$$A_0 + B_{ii}\hat{y}_i\hat{y}_i + B_{i-\tau i+\tau}\hat{y}_{i-\tau}\hat{y}_{i+\tau} + B_{i-2\tau i+2\tau}\hat{y}_{i-2\tau}\hat{y}_{i+2\tau} = 0$$

$$\boxed{\hat{y}_i^2 = A + B\hat{y}_{i-2\tau}\hat{y}_{i+2\tau} + C\hat{y}_{i-\tau}\hat{y}_{i+\tau}}$$

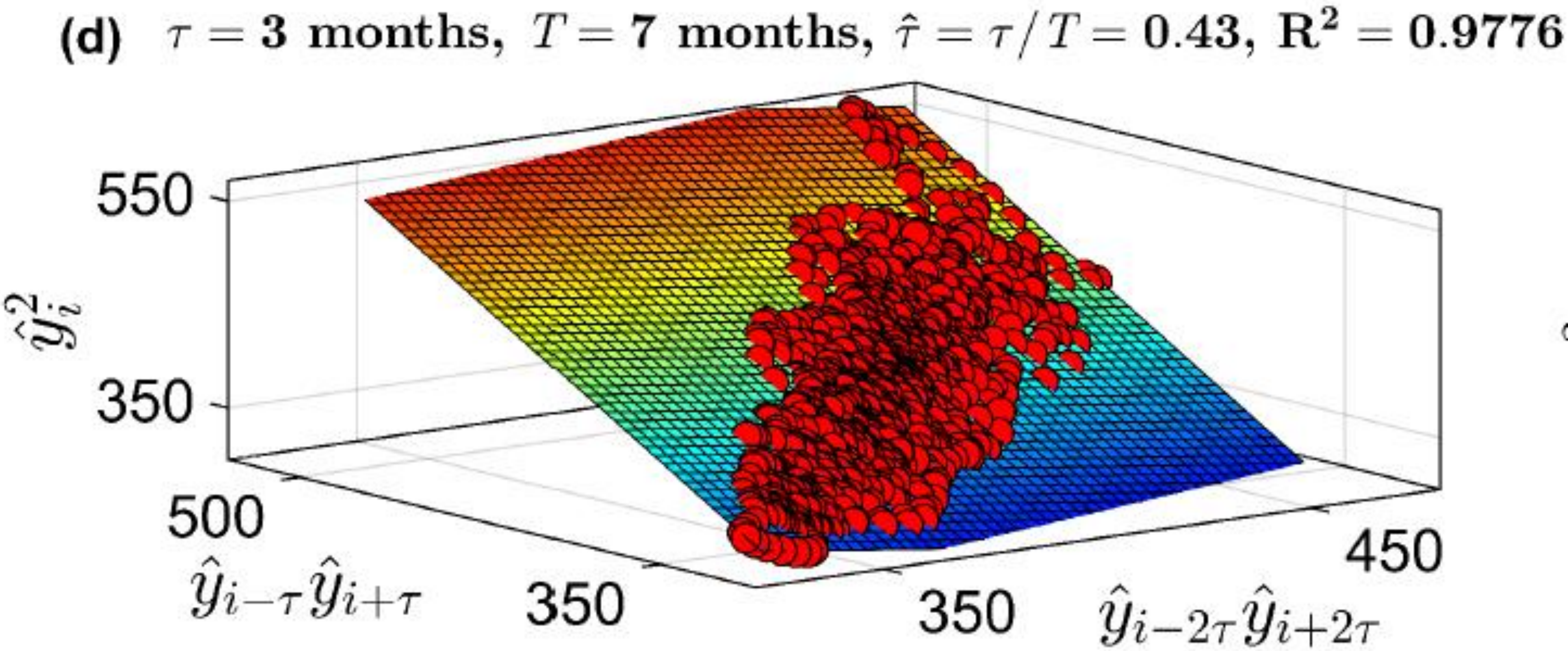
- Nontrivial map
- Works excellently over a range of time delay values $\tau = 1 \text{ month}$ to $\tau = 3 \text{ months}$
- Mapping the 5-dimensional state space to 3-dimensions by nonlinear combinations of state space coordinates

The phenomenological discrete nonlinear model entirely from observations

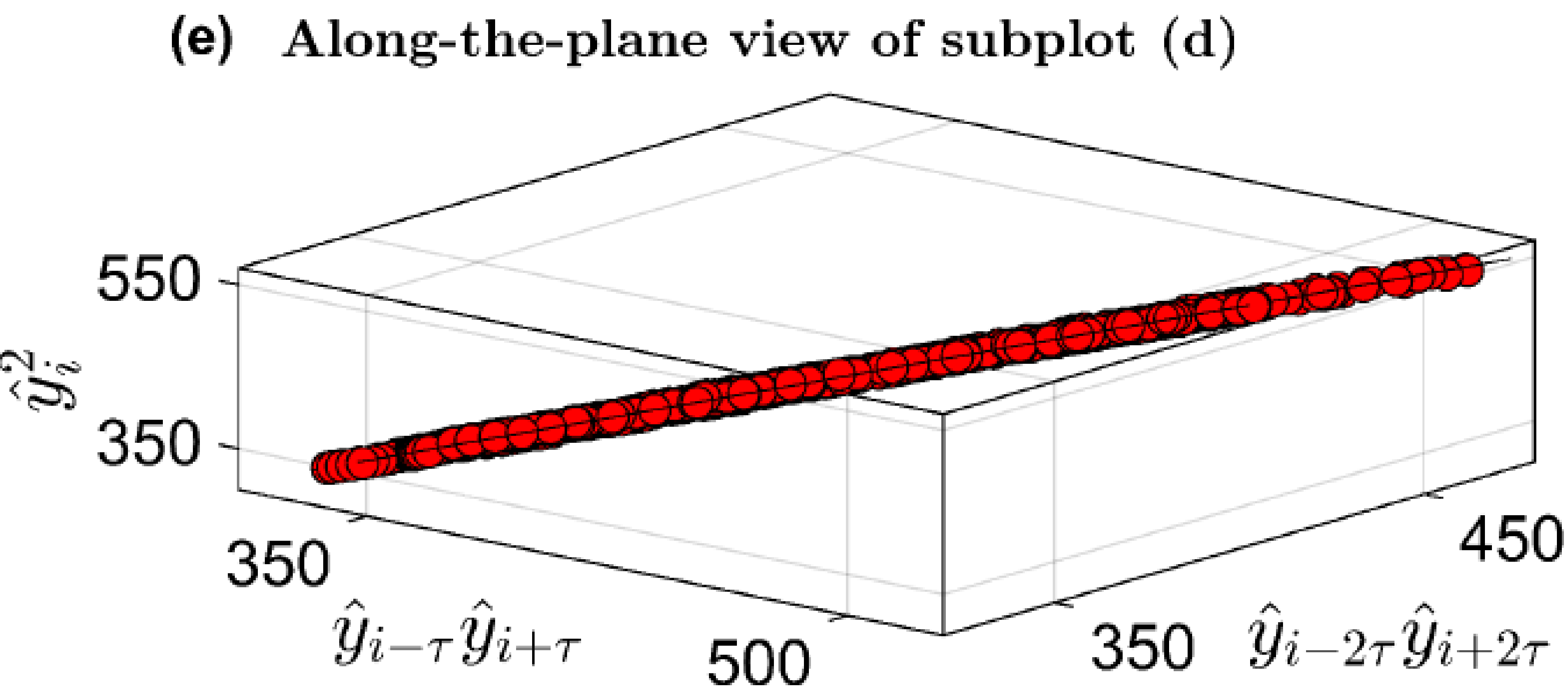


Eureka moment!!

The phenomenological discrete nonlinear model entirely from observations – a closer look



The phenomenological discrete nonlinear model entirely from observations – a closer look



The nonlinear oscillator model

$$\hat{y}_i^2 = A + B\hat{y}_{i-2\tau}\hat{y}_{i+2\tau} + C\hat{y}_{i-\tau}\hat{y}_{i+\tau}$$

Taylor series central difference expansions with step size $\hat{\tau}$

$$\hat{y}_i' = \left(\hat{y}_{i+\tau} - \hat{y}_{i-\tau}\right)/2\hat{\tau}$$

$$\hat{y}_i'' = \left(\hat{y}_{i+\tau} - 2\hat{y}_i + \hat{y}_{i-\tau}\right)/\hat{\tau}^2$$

Square and subtract to obtain $\hat{y}_{i+\tau}\hat{y}_{i-\tau}$ in terms of $\hat{y}_i$ and derivatives at $\hat{y}_i$

Similar exercise with step size $2\hat{\tau}$ yields expression for $\hat{y}_{i+2\tau}\hat{y}_{i-2\tau}$

Valid only till $2\hat{\tau} < 1$

Trajectory out of plane for $\hat{\tau} > 0.43$

The nonlinear oscillator model – an IVP

$$\boxed{\hat{y}_i''\hat{y}_i''' + (P_2/2P_1)\left(\hat{y}_i\hat{y}_i''' - \hat{y}_i'\hat{y}_i''\right) + (P_3/P_1)\left(\hat{y}_i\hat{y}_i'\right) = 0}$$

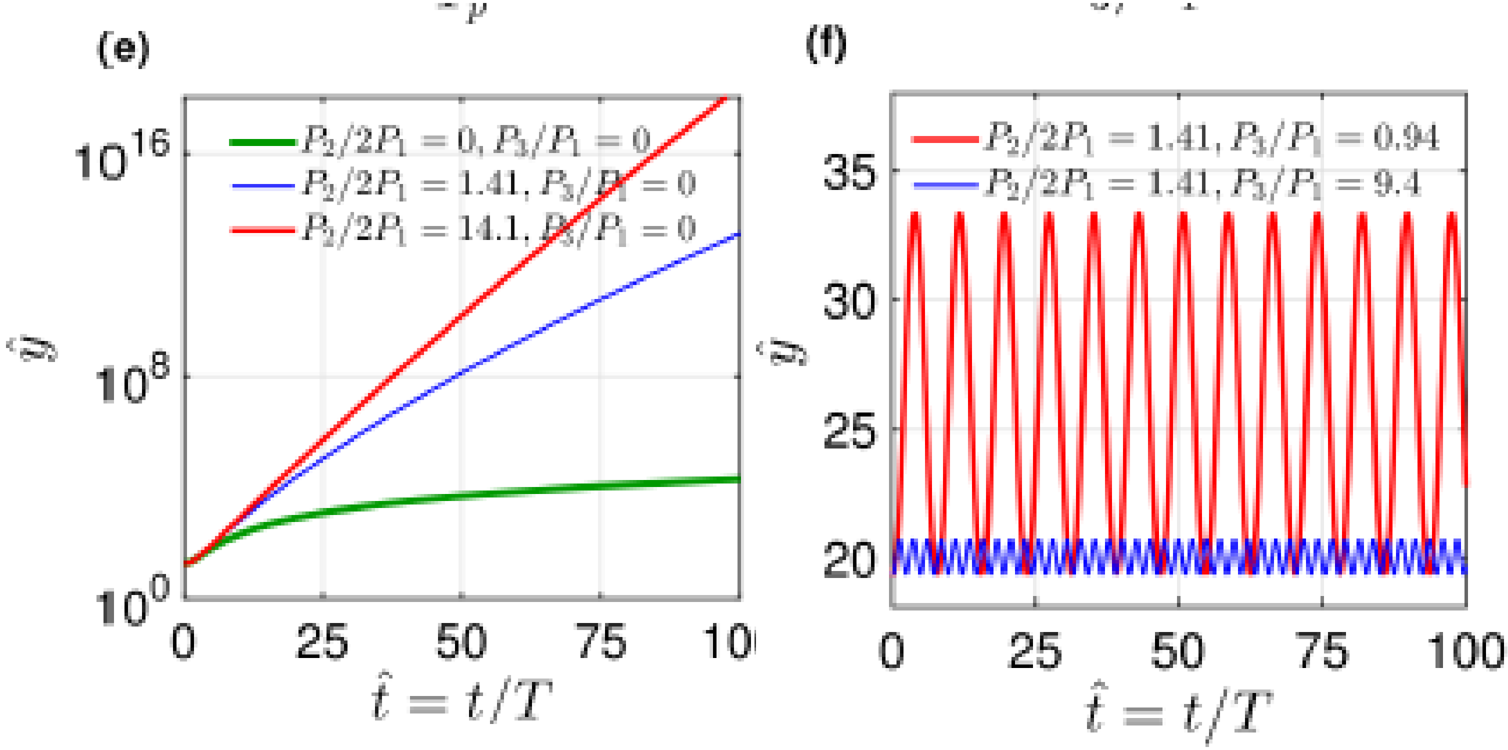
$$P_1 = (4B + C/4)\hat{\tau}^4, \; P_2 = (4B + C)\hat{\tau}^2 \text{ and } P_3 = B + C - 1$$

1. With coefficients obtained from the complete ENSO slow manifold data, Eq. 3 admits a self-sustained, quasi-periodic, non-sinusoidal solution with period of 4.5 years which is in excellent agreement with the dominant periodicity of the ENSO slow manifold.
2. The ENSO slow manifold data show that the coefficients in Eq. 3 can vary with time and this would excite decadal and multidecadal modes in the solution explaining the observed broad spectrum of the ENSO slow manifold.

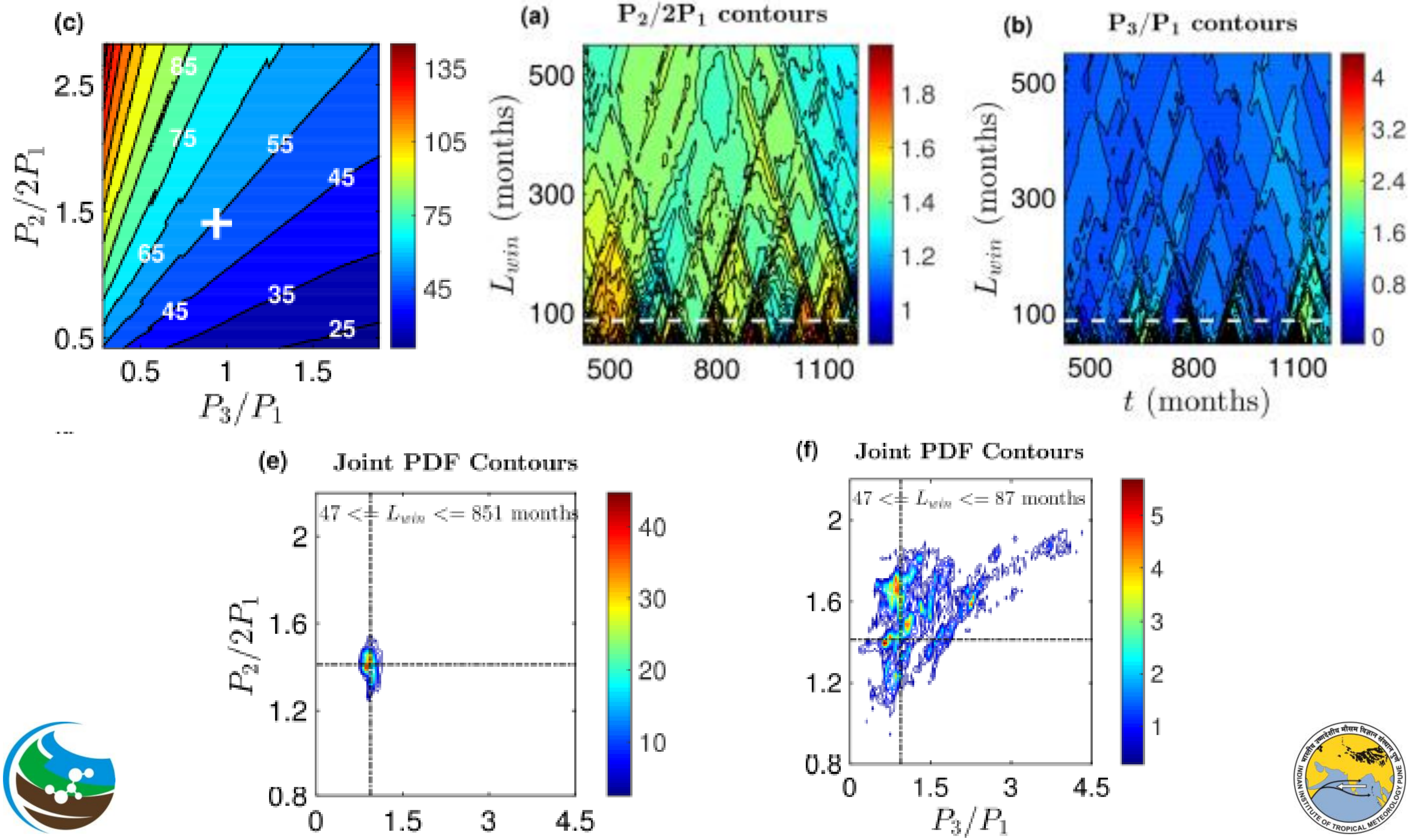
The nonlinear oscillator model – physical significance of terms

$$\boxed{\hat{y}_i''\hat{y}_i''' + (P_2/2P_1)\left(\hat{y}_i\hat{y}_i''' - \hat{y}_i'\hat{y}_i''\right) + (P_3/P_1)\left(\hat{y}_i\hat{y}_i'\right) = 0}$$

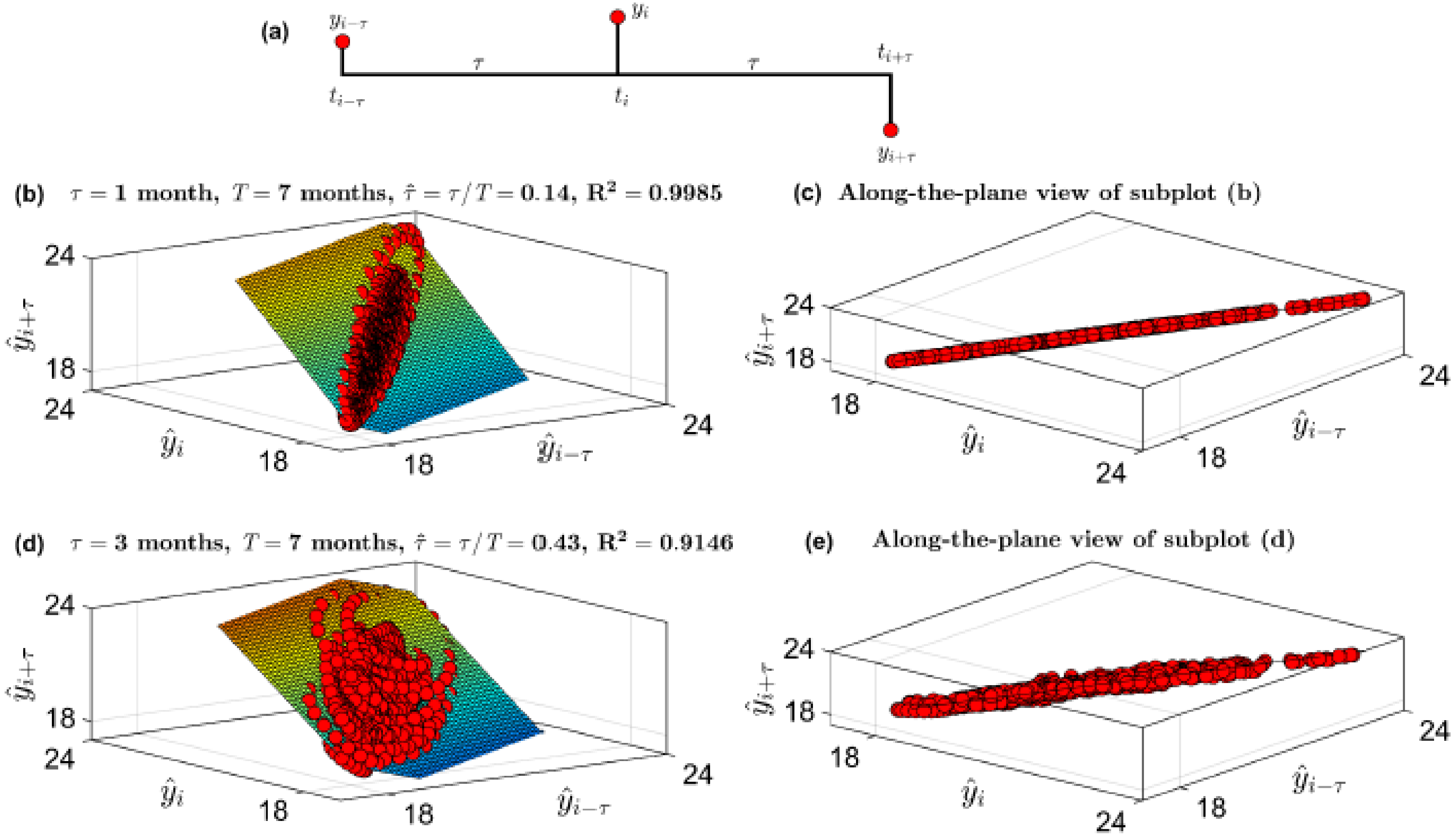
Instability term **Instability term** **Negative Feedback term**



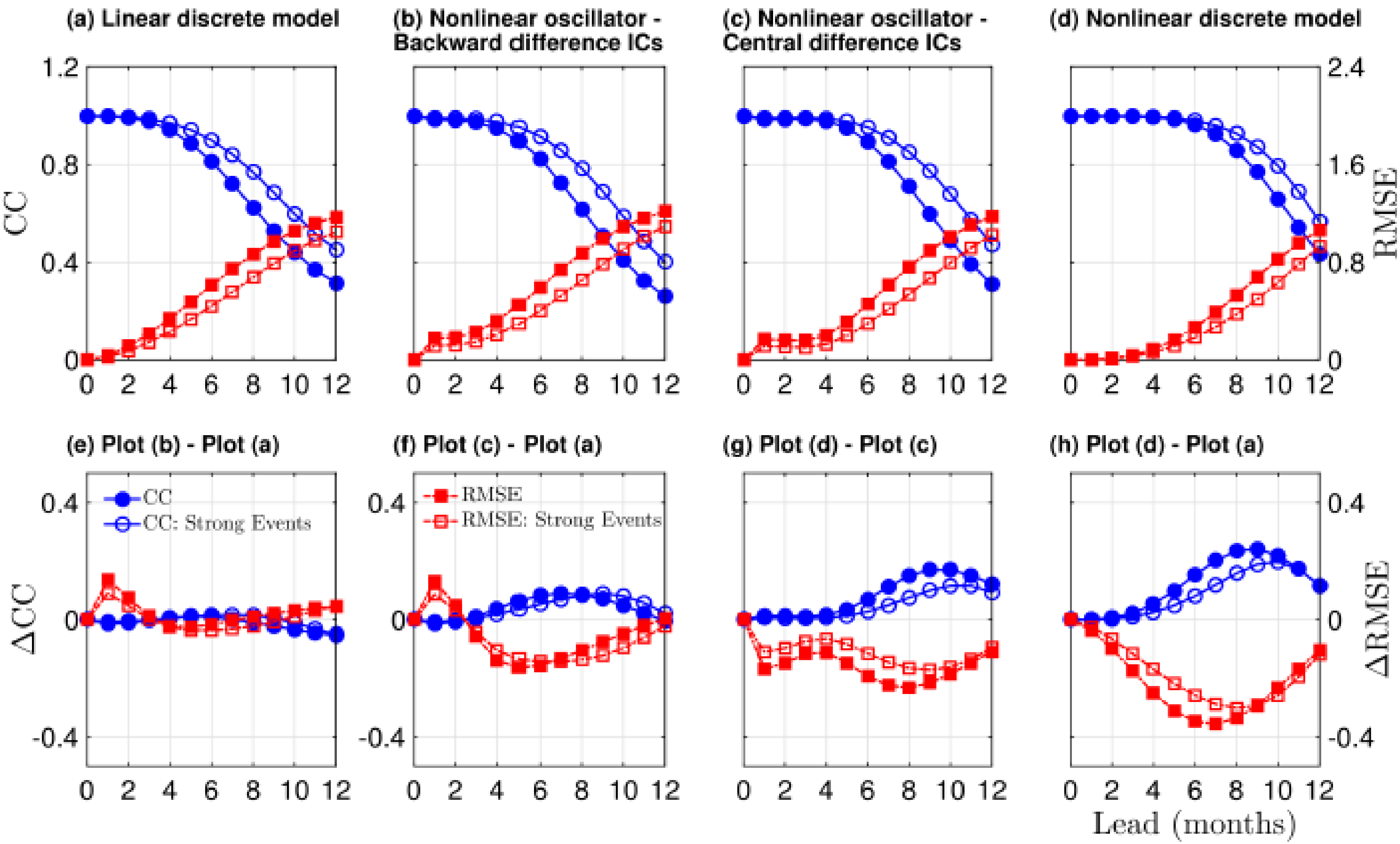
The nonlinear oscillator model – supporting various modes



A discrete "linear" model

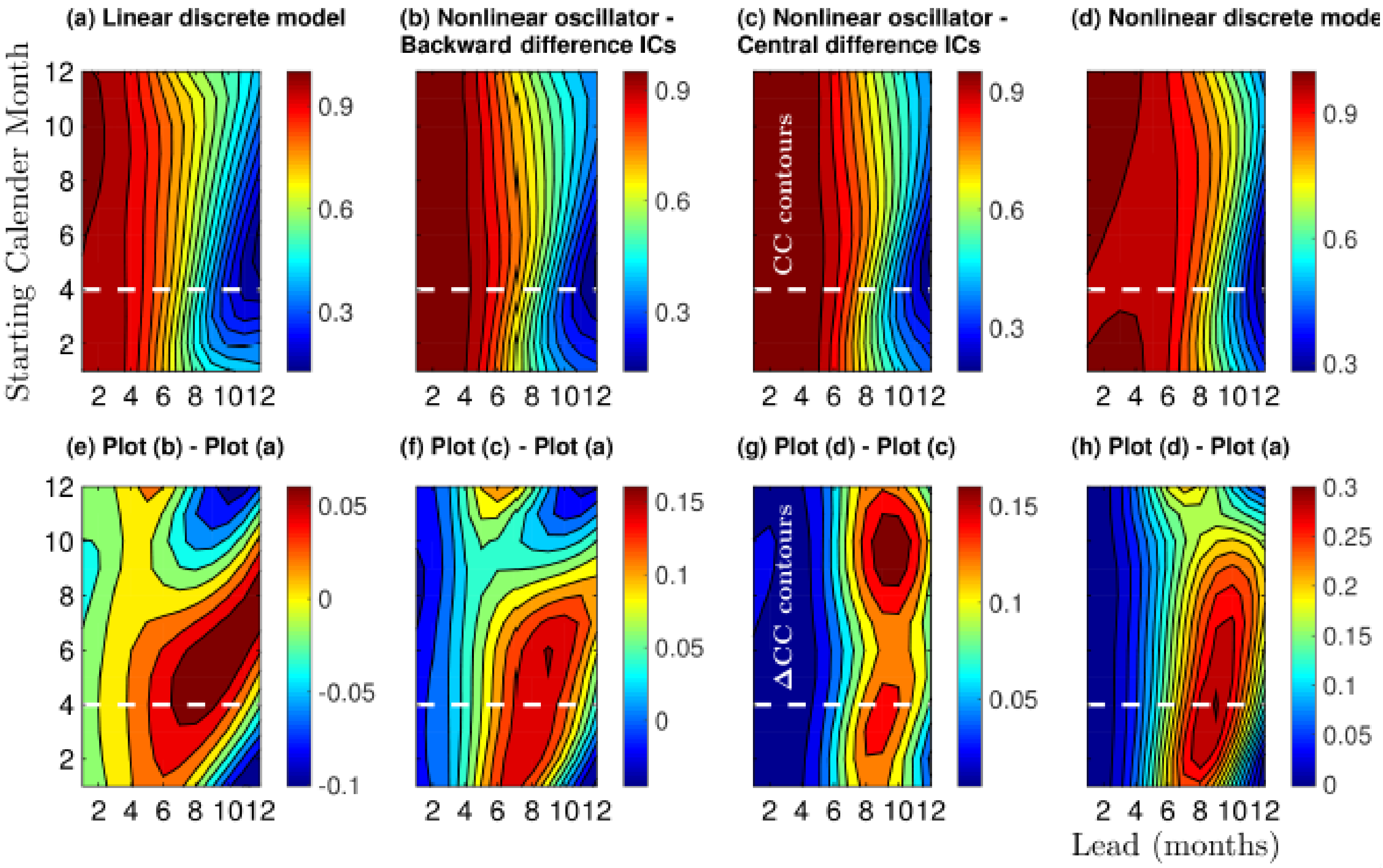


Hindcast Skill Comparisons



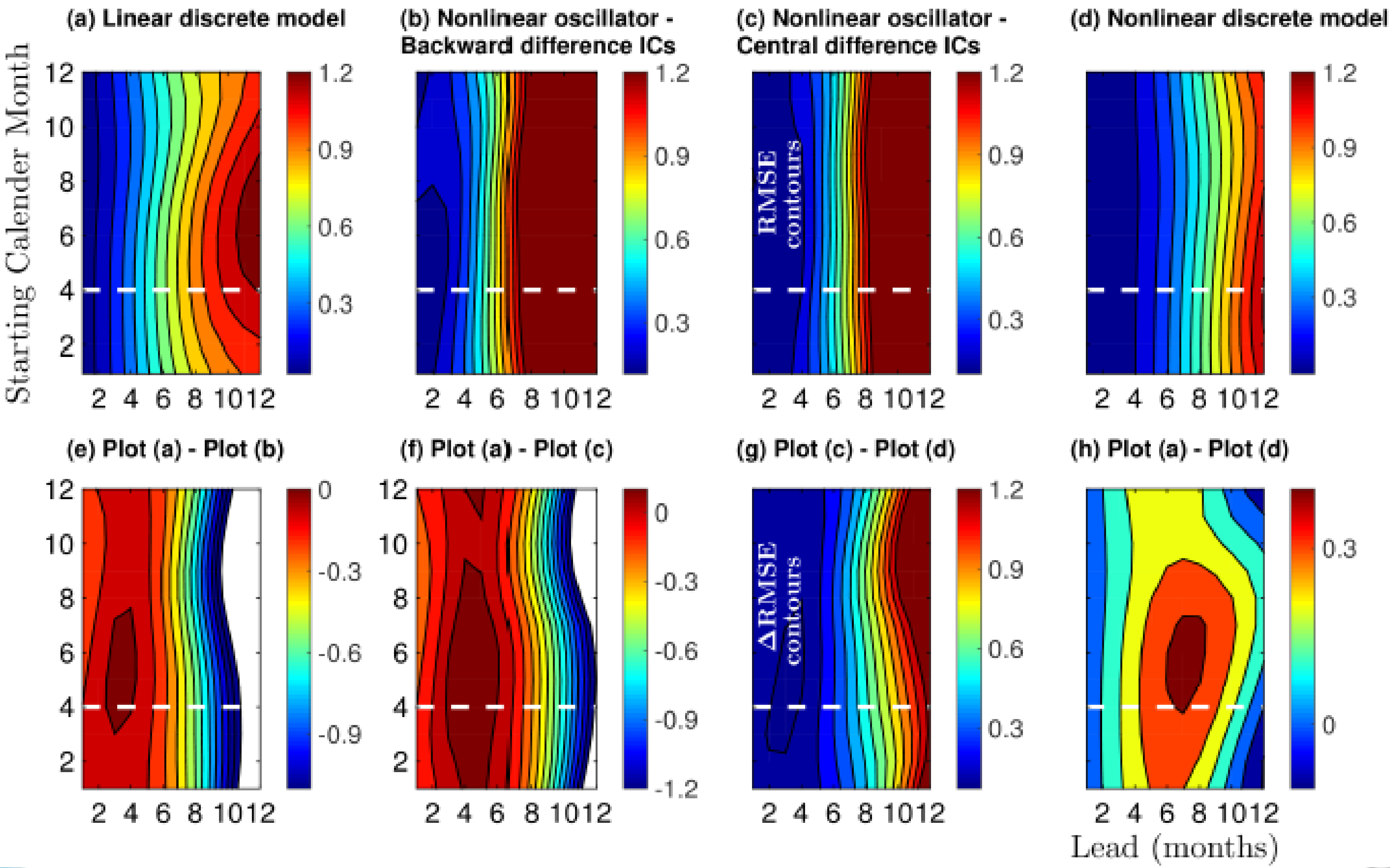
Use 30% of the data points to obtain the map coefficients and then do predictions for the rest 70% points. Nonlinear discrete model performs the best amongst all.

Seasonality of the hindcasts

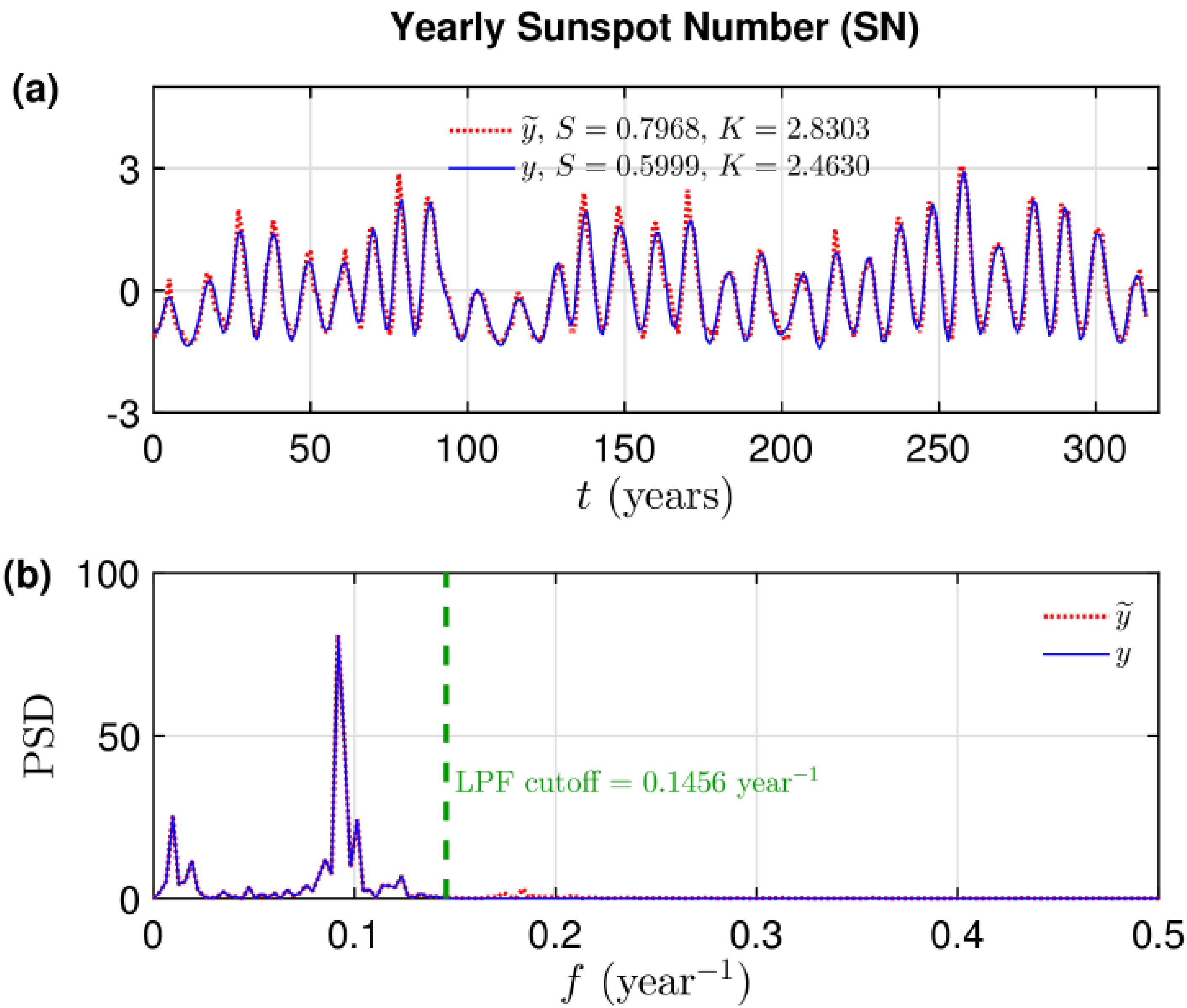


SPB appears to be partly related to the inability to capture correct nonlinearity

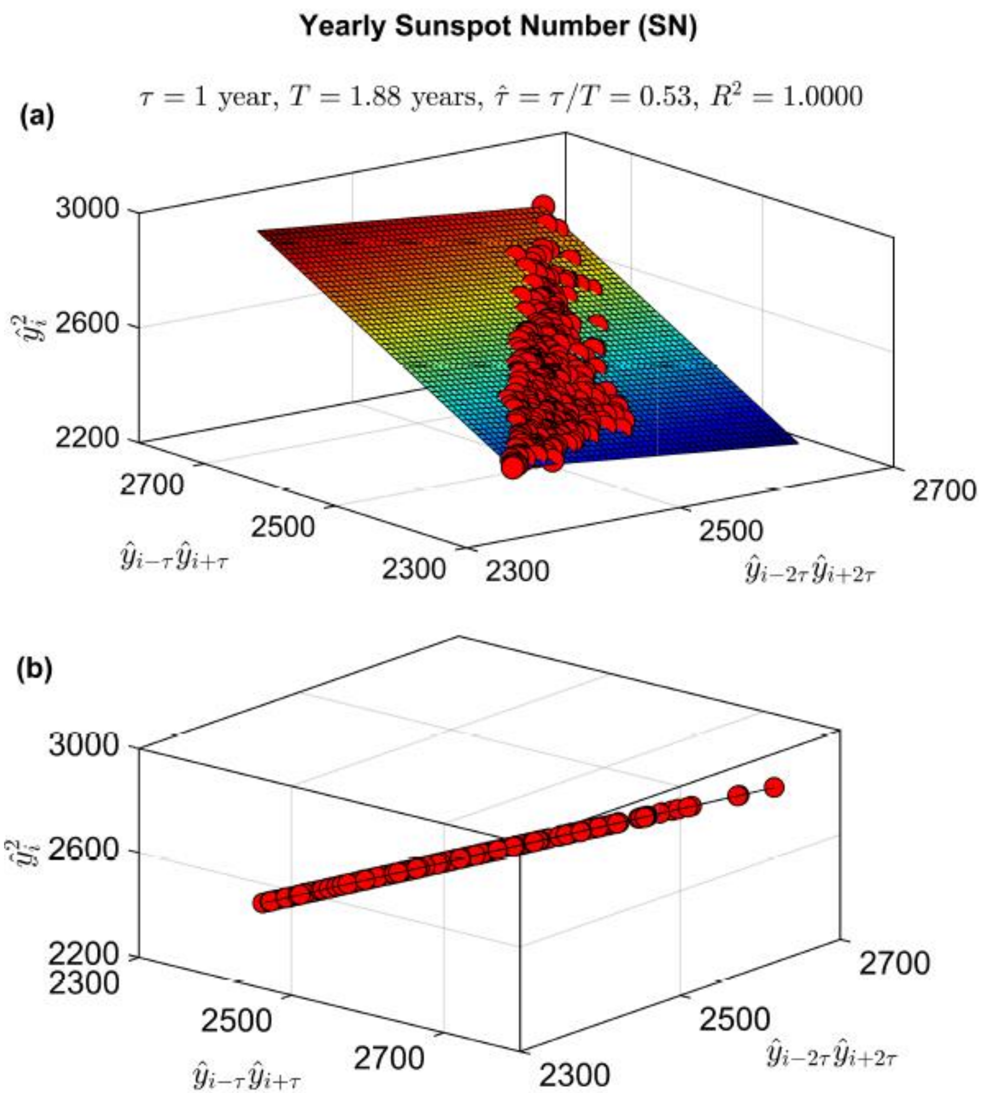
Seasonality of the hindcasts

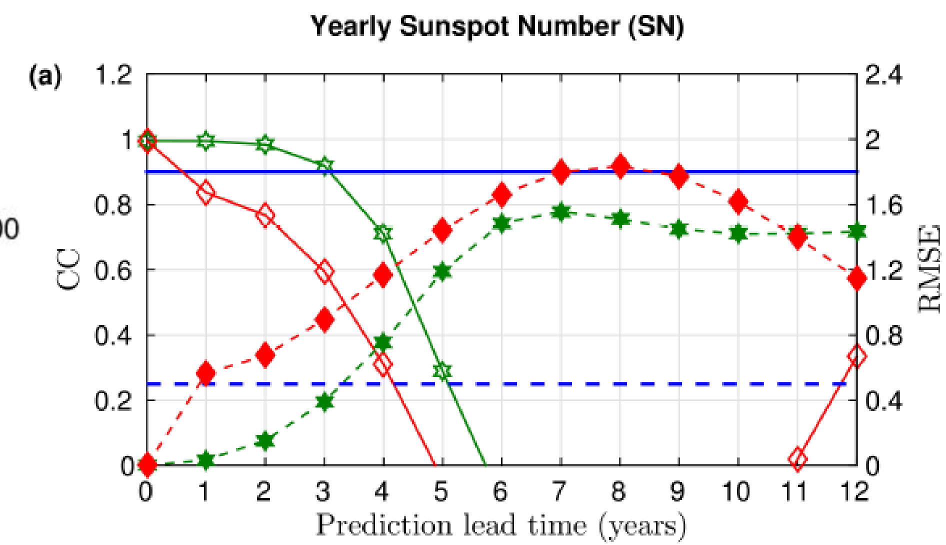


Sunspot number time series



Sunspot number time series … contd.





Conclusions

- A time-invariant, discrete, nonlinear map can be obtained entirely from observations using nonlinear dynamics ideas. A less accurate linear map can also be obtained.

- The nonlinear map distills into a nonlinear ODE that is consistent with the physical processes responsible for occurrence of ENSO.

- Hindcast predictions can be made using these maps and solving the ODE as an initial value problem for the slow manifold of ENSO.

- Hindcast skill of the nonlinear discrete model is the best amongst these since it is free from the finite difference errors in the ODE as well as initial conditions.

Conclusions … contd.

• SPB, at least partly, appears to be related to incorrect capture of nonlinearity along with (in)accuracy of the initial conditions.

• Real-time predictions are more challenging since the filtered climate noise affects the slow manifold towards the most recent observations – the so-called "end effect". We are working on methods that can address these issues.

• The map appears to have universal character across quasi-periodic phenomena ranging from ENSO, Sunspot number and GIG (not shown) time series data.

Acknowledgements

- Director, Prof. Ravi Nanjundiah

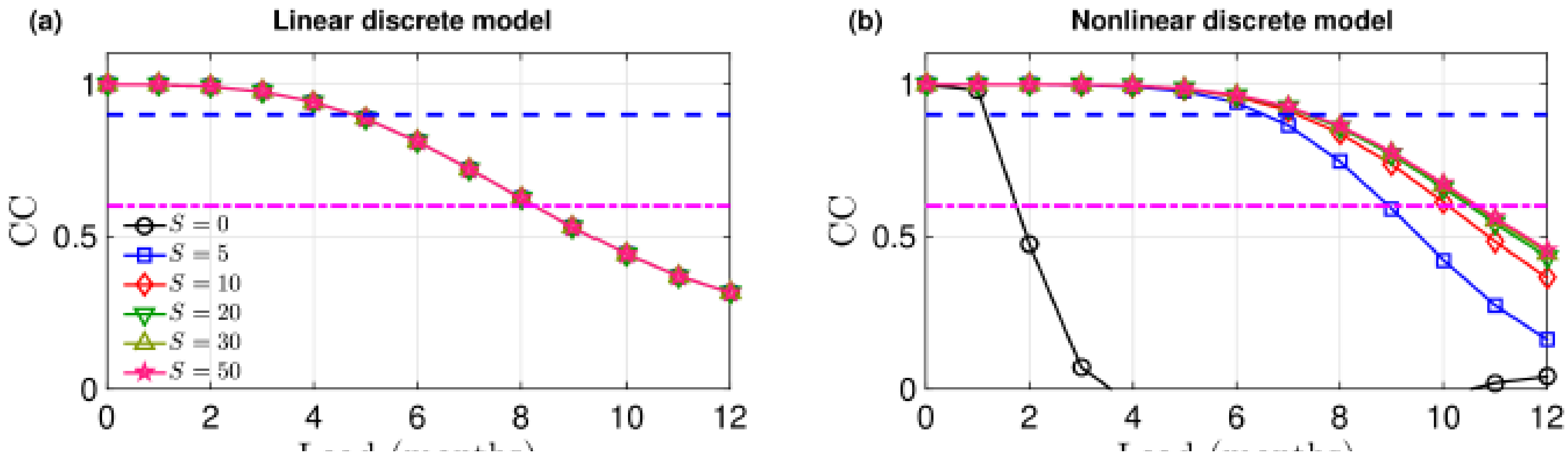
Indian Institute of Tropical Meteorology (IITM), Pune

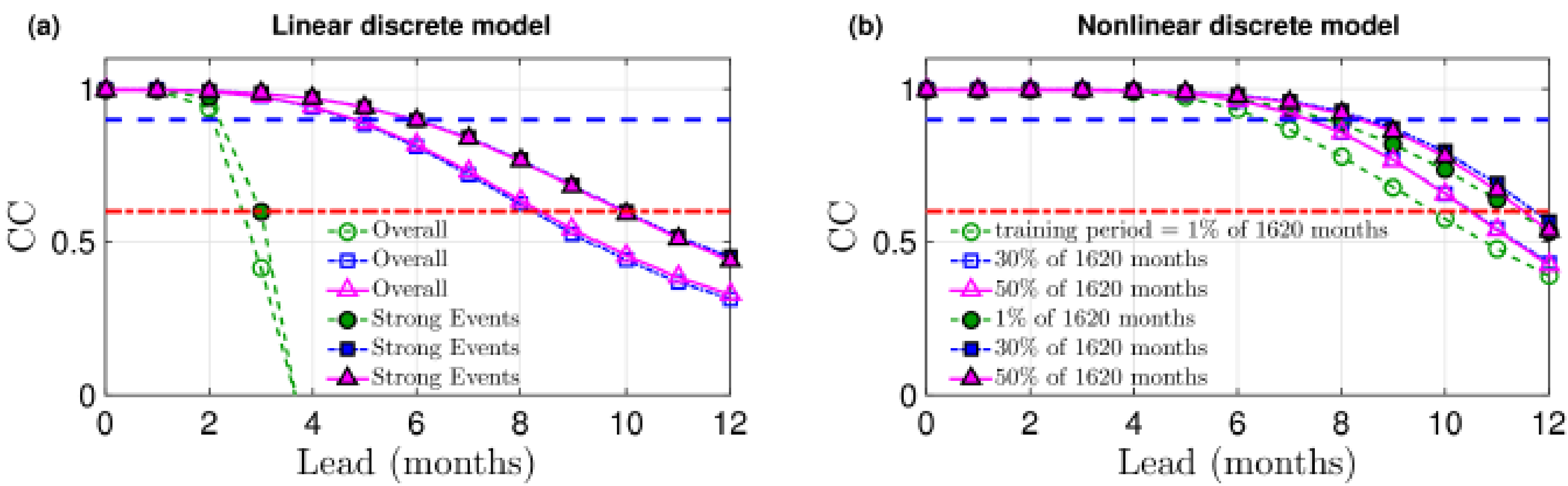
- Dr. Thara Prabhakaran, Project Director, CAIPEEX, IITM, Pune
- Ministry of Earth Sciences (MoES), Government of India
- Prof. R. Narasimha (JNCASR, Bengaluru), Prof. O. N. Ramesh (IISc, Bengaluru), Prof. G. S. Bhat (IISc, Bengaluru), Prof. G. Ambika (IISER, Pune), Mr. Rahul Chopde (Scientist, Defense Research Development Organization)
- Mr. Harish Choudhary, Mr. Abhishek Gupta (my students!!)
- Prof. B. N. Goswami thanks the Science Education and Research Board, India for the Distinguished Scientist Fellowship

Thank You

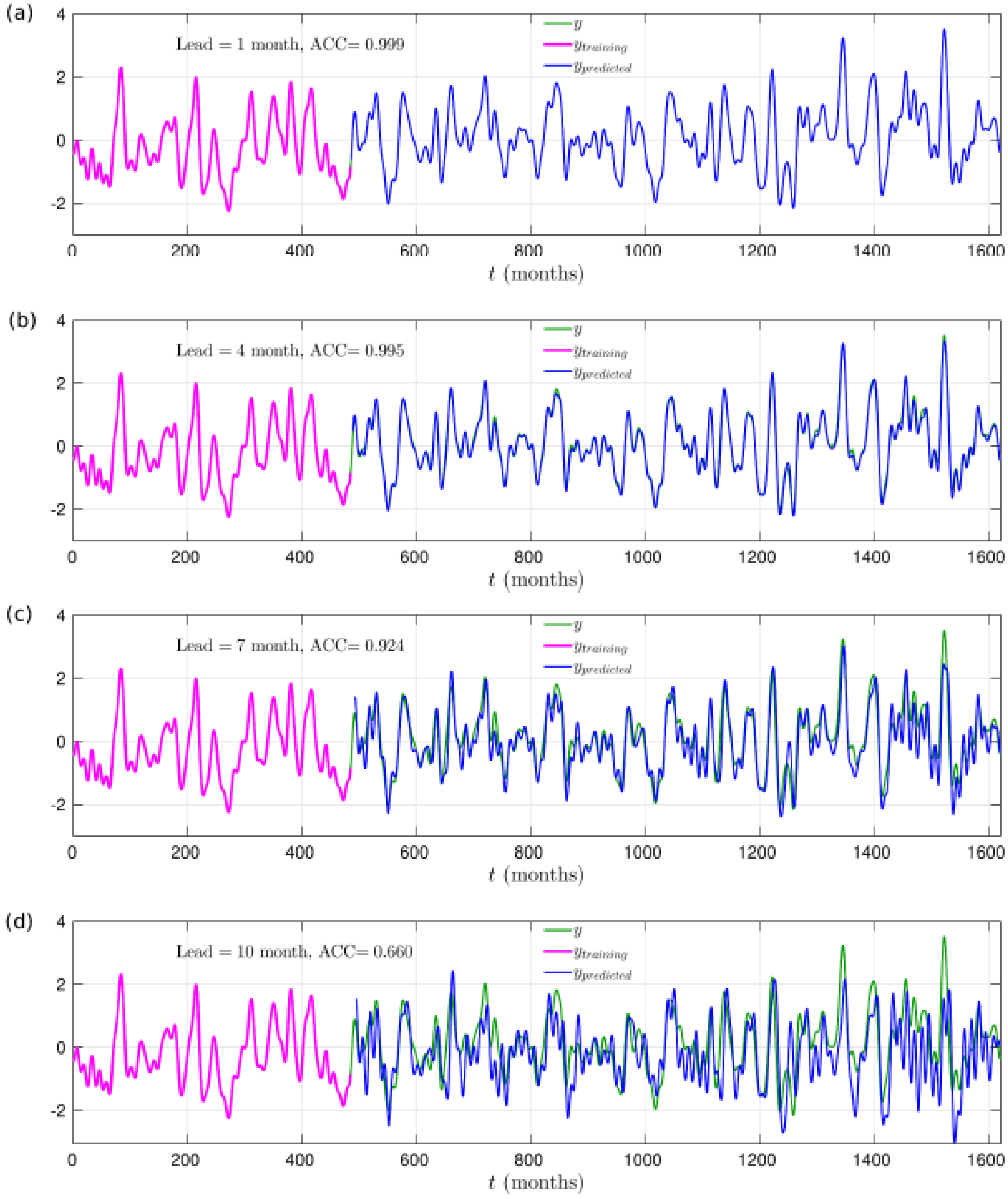
Supplementary Material

Sensitivity studies

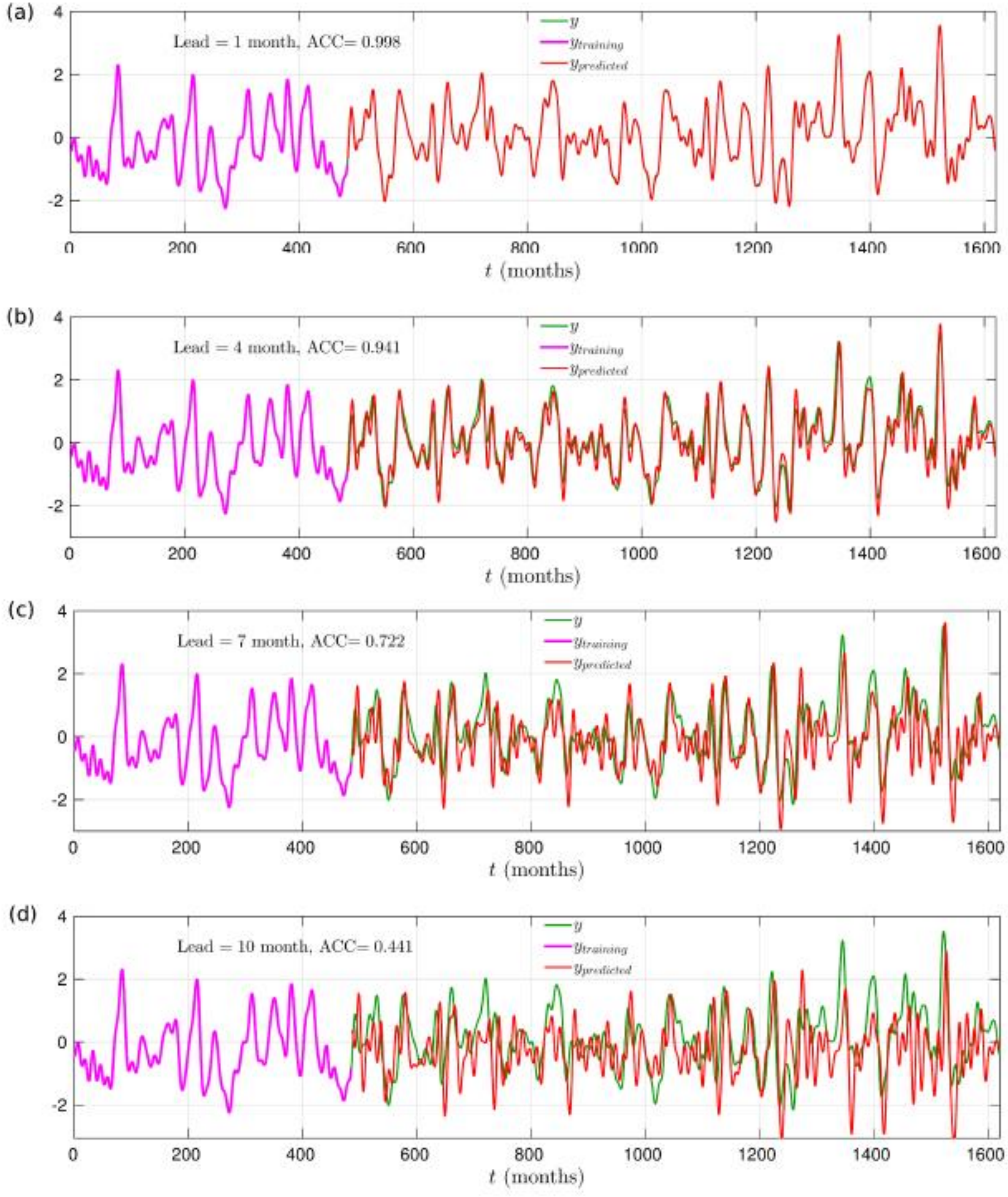




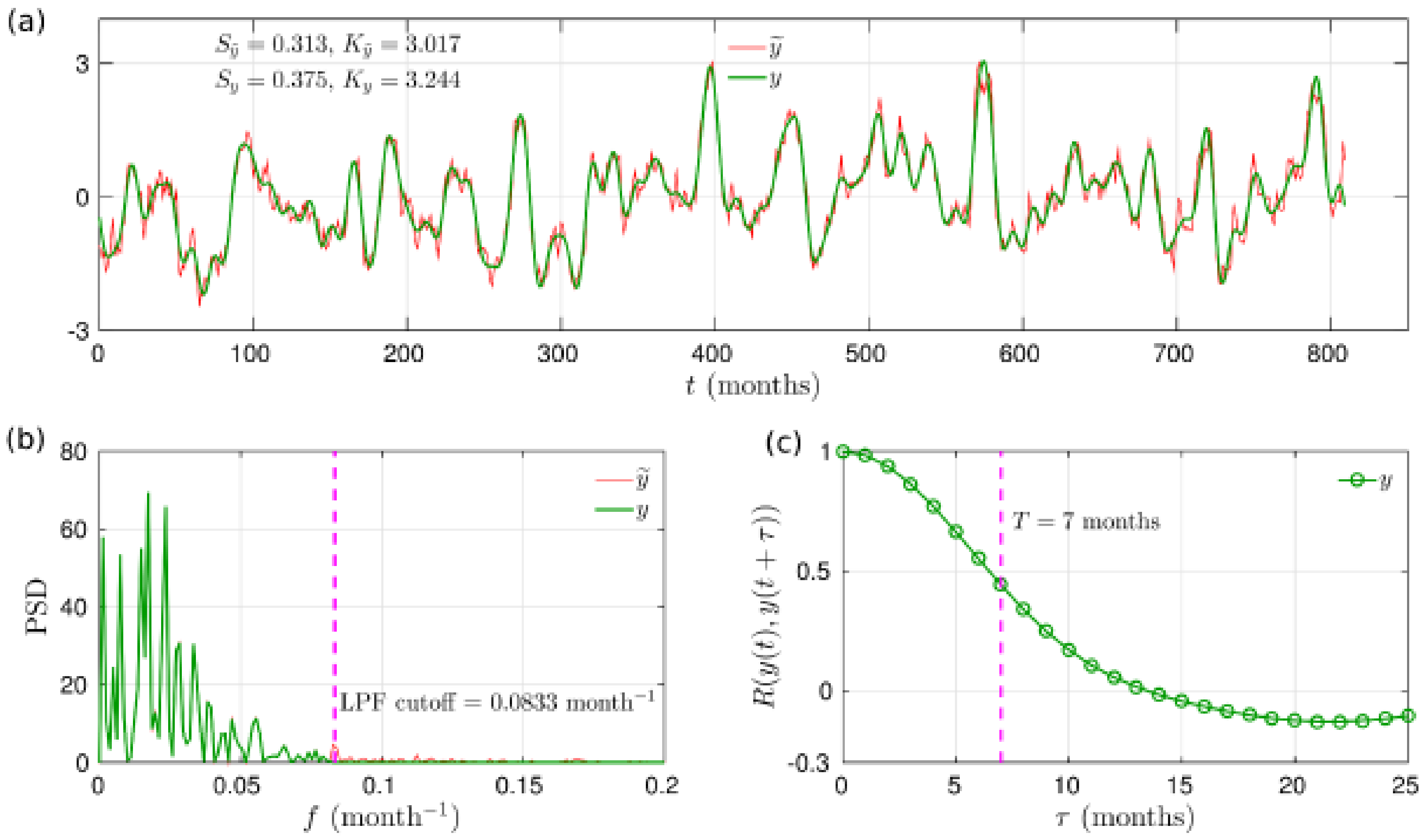
Nonlinear model sample timeseries



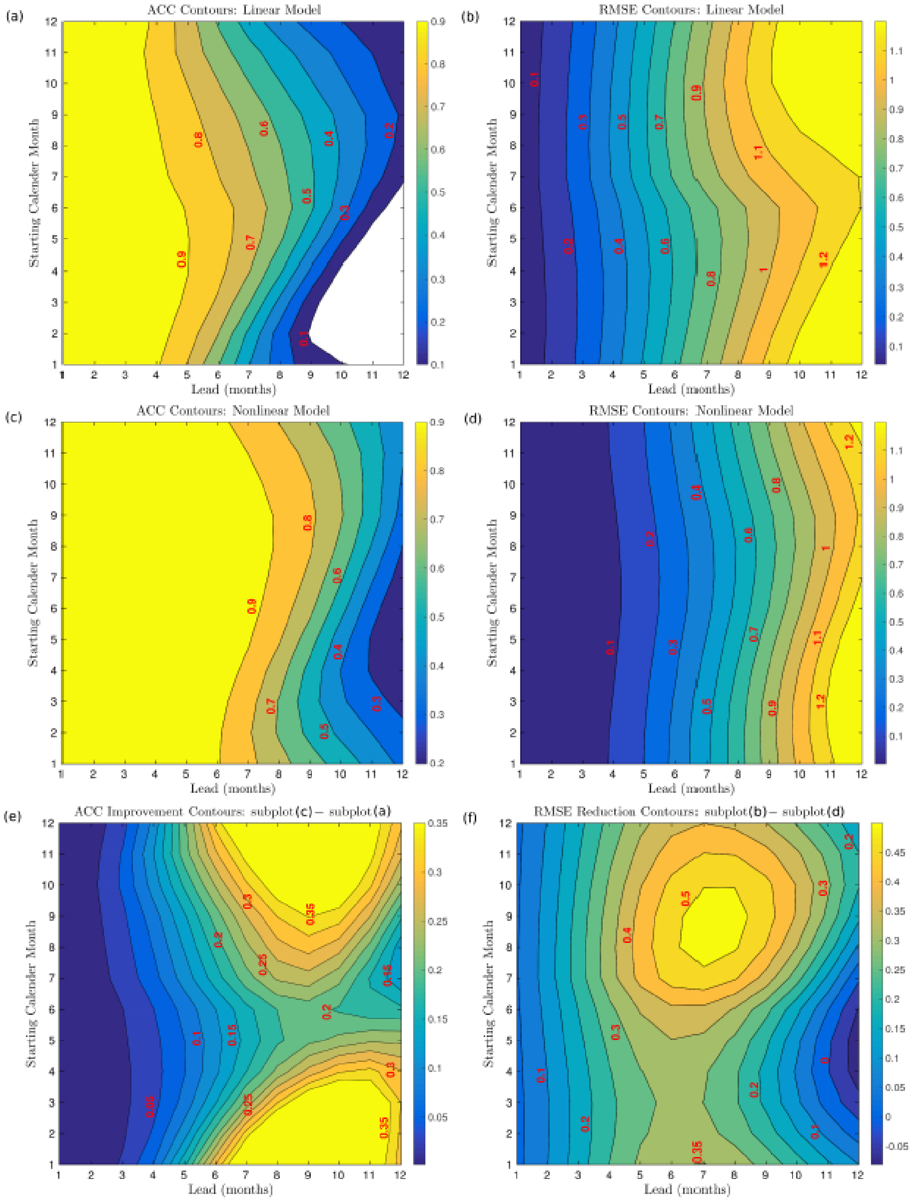
Linear model sample timeseries



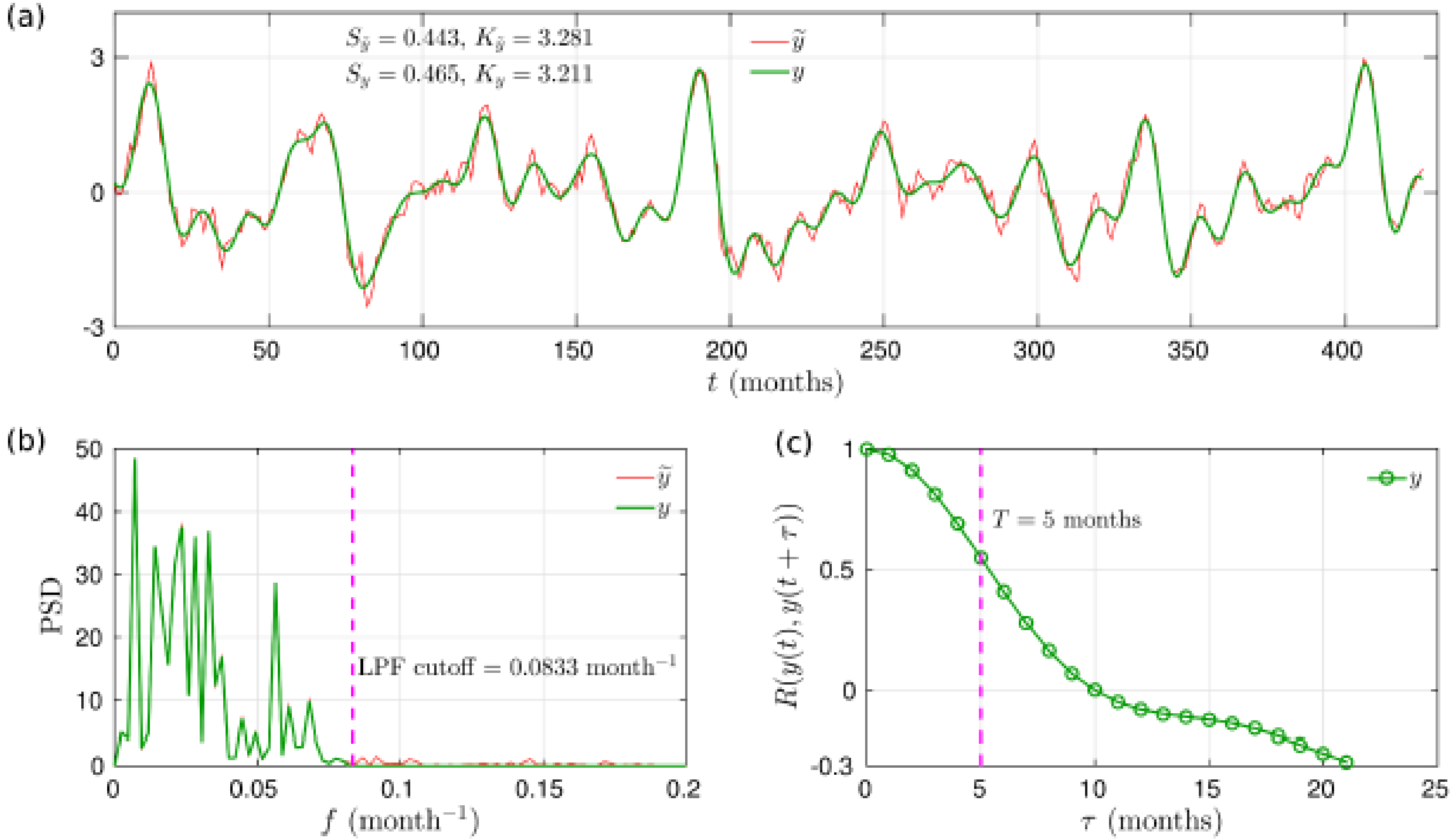
MEI


(a)
$S_{\tilde{y}} = 0.313,\ K_{\tilde{y}} = 3.017$
$S_y = 0.375,\ K_y = 3.244$
$\tilde{y}$
y
3
0
-3
0
100
200
300
400
500
600
700
800
t (months)
(b)
PSD
80
60
40
20
0
0.05
0.1
0.15
0.2
LPF cutoff = 0.0833 month^{-1}
f (month^{-1})
(c)
$R(y(t), y(t+\tau))$
1
0.5
-0.3
$T = 7$ months
5
10
15
20
25
τ (months)

MEI

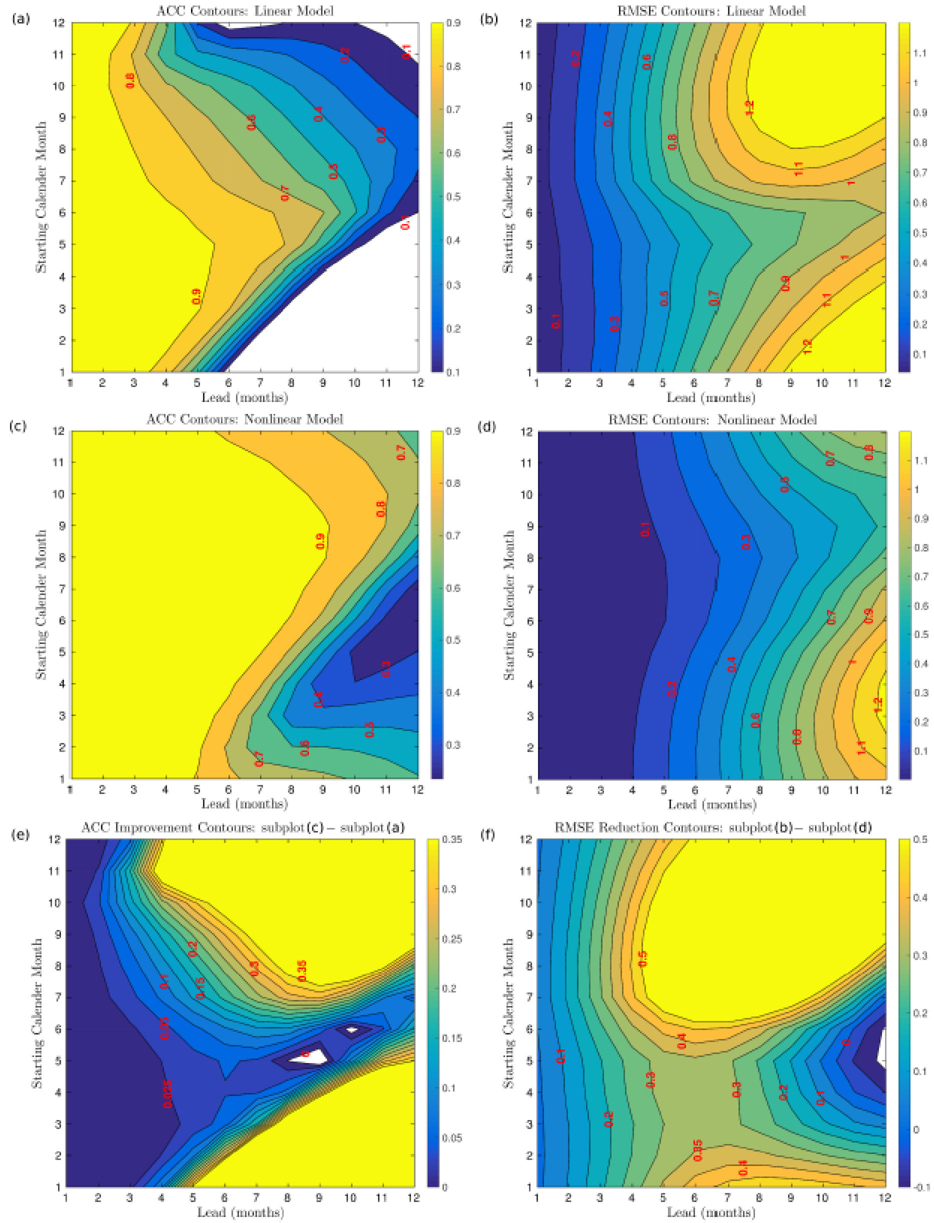

(a)
ACC Contours: Linear Model
(b)
RMSE Contours: Linear Model
(c)
ACC Contours: Nonlinear Model
(d)
RMSE Contours: Nonlinear Model
(e)
ACC Improvement Contours: subplot(c) – subplot(a)
(f)
RMSE Reduction Contours: subplot(b) – subplot(d)
Starting Calender Month
Lead (months)

Nino3.4



(a)
$S_{\tilde{y}} = 0.443,\ K_{\tilde{y}} = 3.281$
$S_y = 0.465,\ K_y = 3.211$
$\tilde{y}$
y
t (months)
(b)
PSD
LPF cutoff = 0.0833 month^{-1}
f (month^{-1})
(c)
$T = 5$ months
$R(y(t), y(t+\tau))$
τ (months)

Nino3.4


(a)
ACC Contours: Linear Model
(b)
RMSE Contours: Linear Model
(c)
ACC Contours: Nonlinear Model
(d)
RMSE Contours: Nonlinear Model
(e)
ACC Improvement Contours: subplot(c) – subplot(a)
(f)
RMSE Reduction Contours: subplot(b) – subplot(d)
Starting Calender Month
Lead (months)